Algebra 2 Unit 1: Quadratics

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

Content Standards:

- A.SSE.1. Interpret expressions that represent a quantity in terms of its context.
- A.SSE.1a. Interpret parts of an expression, such as terms, factors, and coefficients.
- A.SSE.2. Use the structure of an expression to identify ways to rewrite it.
- A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
  - A.SSE.3a. Factor a quadratic expression to reveal the zeros of the function it defines.
  - A.SSE.b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- A.APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A.APR.2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on

Transfer

Students will be able to independently use their learning ***compare linear, exponential and quadratic relationships and models in financial, scientific and other data.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- parent functions transform graphically when changes are made to the algebraic function
- transformations can be described through the visual representation of a graph
- systems of three equations with three variables exist with the intersection of more than one plane

ESSENTIAL QUESTIONS

- How do the graphs of y = f(x) + k, y = f(x − h), and y = −f(x) compare to the graph of the parent function f ?
- How can you determine the number of solutions of a linear system?

Acquisition

Students will know…

- that the discriminant of a quadratic determines the number and type of solution.
- that you find the average rate of change by calculating the slope for multiple type of functions
- that the standard form of a quadratic is in the form of $f(x) = ax^2 + bx + c$
- that there are multiple forms that a quadratic may be presented in, including intercept form, vertex form and standard form.
- that the root of equation is another way to state the solutions or to find the x-intercepts
- that the zero of a function are the point that the line of the graph crosses the x-axis and that y is equal to 0
- that there are multiple ways to solve a quadratic function including, graphing, completing the square, square root property, factoring and the quadratic formula

Students will be skilled at…

- interpreting expressions that represent a quantity in terms of its context.
- interpreting parts of an expression, such as terms, factors, and coefficients.
- using the structure of an expression to identify ways to rewrite it.
- choosing and producing an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
- factoring a quadratic expression to reveal the zeros of the function it defines.
- completing the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- recognizing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). ● A.APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. ● A.APR.4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples. ● A.APR.5. (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. ● A.APR.6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. ● A.APR.7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add,subtract, multiply, and divide rational expressions. ● A.CED.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ● A.CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. ● A.REI.4. Solve quadratic equations in one variable. o A.REI.4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. o A.REI.4b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the	● that with each method there is a better choice for solving the quadratic depending on the quadratic that is presented. ● that the properties used for solving linear systems of equations may be used to help solve non-linear systems of equations including, graphing, substitution, and elimination ● that a quadratic inequalities are presented in one or two variables, how to solve algebraically and how to graph each type of inequality ● what the imaginary unit is, how to perform operations using the imaginary unit and how to apply algebraic properties when working with imaginary units and complex numbers. often needed when working with complex numbers. ● how to identify families of functions, describe transformations of parent functions, and write functions representing combinations of transformations ● how to find the lines of fit and lines of best fit and how to write a prediction equation ● that the algebraic properties are used to solve systems of linear equations in three variables ***vocabulary:*** function, domain, range, average rate of change, systems of three linear equations, ordered triple, matrix, parabola, vertex, axis of symmetry, maximum value, minimum value, imaginary unit, complex number, imaginary number, pure imaginary, completing the square, perfect square trinomial, discriminant, focus, diretrix	● knowing and applying the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). ● identifying zeros of polynomials when suitable factorizations are available, and using the zeros to construct a rough graph of the function defined by the polynomial. ● proving polynomial identities and use them to describe numerical relationships. ● knowing and applying the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. ● rewriting simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. ● recognizing that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add,subtract, multiply, and divide rational expressions. ● creating equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ● rearranging formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. ● solving quadratic equations in one variable. ● using the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. ● deriving the quadratic formula from this form.

quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. ● A.REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). ● A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ ● F.IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). ● F.IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. ● F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ ● F.IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.		● solving quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. ● recognizing when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

● F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ ○ F.IF.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima. ○ F.IF.7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ○ F.IF.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ○ F.IF.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ○ F.IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ● F.IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. ○ F.IF.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. ● F.BF.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. ● F.BF.4. Find inverse functions. ○ F.BF.4a. Solve an equation of the form f(x) = c for a simple function f that has an		

inverse and write an expression for the inverse. ● F.TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★ ● G.GPE.2. Derive the equation of a parabola given a focus and directrix. ● S.ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ○ S.ID.6a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.		
Content Area Literacy Standards		***21st Century Skills***
● RH 9-10.7 Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem. ● RST 9-10.4 Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context ● RST. 9-10.3 Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text. ● WHST.9-10.1 Write arguments focused on *discipline-specific content*. ● WHST.9-10.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.		● ***Solve Problems*** ● ***Communicate clearly*** ● ***Collaborate with others*** ● ***Be self-directed learners*** ● ***Reason effectively***

Algebra 2 Unit 2: Higher Order Functions

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

Content Standards:

- A.SSE.1. Interpret expressions that represent a quantity in terms of its context.
  - A.SSE.1a. Interpret parts of an expression, such as terms, factors, and coefficients.
- A.SSE.2. Use the structure of an expression to identify ways to rewrite it.
- A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  - A.SSE.3a. Factor a quadratic expression to reveal the zeros of the function it defines.
  - A.SSE.3c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
- A.APR.1. Understand that polynomials form a system analogous to the integers, namely, they

Transfer

Students will be able to independently use their learning to ***model relationships among quantities, find a solution and evaluate the reasonableness of that solution.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- property of numbers and functions extends deeper into the study of Algebra
- all functions can be visually interpreted through graphical representations
- functions can be used to demonstrate real world situations

ESSENTIAL QUESTIONS

- How do we use different functions to model real world problems and their solutions?
- How are the number properties learned in earlier math courses applied at a deeper level in Algebra 2?

Acquisition

Students will know…

- that the Remainder Theorem says for a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
- that properties of mathematics are applied to polynomials using addition, subtraction, multiplication and division.
- that solutions of polynomials and the zeros of the polynomial functions are at the x-intercepts
- that polynomial quotients can be found using long division or synthetic division.
- that properties of mathematics are applied through the process of factoring.
- that the factor theorem is used with division of polynomials.
- that conjugate values are used for simplifying rational expressions.

Students will be skilled at…

- interpreting expressions that represent a quantity in terms of its context.
- interpreting parts of an expression, such as terms, factors, and coefficients.
- using the structure of an expression to identify ways to rewrite it.
- choosing and producing an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- factoring a quadratic expression to reveal the zeros of the function it defines.
- using the properties of exponents to transform expressions for exponential functions.
- recognizing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.

are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. ● A.APR.2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). ● A.APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. ● A.APR.4. Prove polynomial identities and use them to describe numerical relationships. ● A.APR.6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. ● A.APR.7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. ● A.CED.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ● A.CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. ● A.REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. ● A.REI.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). ● A.REI.10. Understand that the graph of an equation in two variables is the set of all its	● that properties of rational exponents are used to simplify expressions with rational exponents and to write in simplest form. ● show the graph of radical functions and the transformation of the radical functions is different from the parent function ● that mathematical properties (add, subtract, multiply and divide) are used with radical equations and inequalities. ● that the mathematical properties of logarithms and the change-of-base formula are used to define, evaluate logarithms, and solve logarithms ● how to graph and transform the logarithm function and the transformation ● that the mathematical properties are used to add, subtract, multiply and divide rational expressions to simplify and solve ● that trigonometric functions are used to find unknown side lengths and angle measures of angles, including right triangles ● that the sum and difference formulas are used to evaluate and simplify trigonometric expressions and to solve trigonometric equations ***vocabulary:*** Polynomial, Polynomial function,End behavior, Pascal's Triangle, Polynomial long division, Synthetic division, Factored completely, Factor by grouping, Quadratic form, Repeated solution, Complex conjugates, Local maximum, Local minimum, Even function, Odd function, Finite differences, nth root of a, Index, Simplest form, Conjugates, Like radicals, Radical function, Radical equations, Extraneous solutions, Inverse functions, Exponential function, Exponential growth function, Growth factor, Asymptote, Exponential decay function, Decay factor, Natural base e, or the Euler number, Logarithm of y with base b, Common logarithm, Natural logarithm, Exponential equations, Logarithmic equations, Inverse variation, Constant of variation, Rational function, Simplified form, Complex fraction, Cross multiplying, Sine, Cosine, Tangent, Cosecant, Secant, Cotangent, Initial side, Terminal side, Standard position, Coterminal, Radian, Sector, Central angle, Unit circle, Quadrantal angle, Reference angle, Amplitude,	● adding, subtracting, and multiplying polynomials. ● knowing and applying the Remainder Theorem. ● identifying zeros of polynomials when suitable factorizations are available. ● using the zeros to construct a rough graph of the function defined by the polynomial. ● proving polynomial identities. ● using them to describe numerical relationships. ● rewriting simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. ● recognizing that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. ● adding, subtracting, multiplying, and dividing rational expressions. ● creating equations and inequalities in one variable and using them to solve problems. Including equations arising from linear and quadratic functions, and simple rational and exponential functions. ● representing constraints by equations or inequalities, and by systems of equations and/or inequalities. ● interpreting solutions as viable or nonviable options in a modeling context. ● solving simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

solutions plotted in the coordinate plane, often forming a curve (which could be a line). ● A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ ● F.IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). ● F.IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. ● F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ ● F.IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ● F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ ○ F.IF.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	Periodic function, Cycle, Period, Phase shift, Midline, Frequency, Sinusoids, Trigonometric identity	

○ F.IF.7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ○ F.IF.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ○ F.IF.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ○ F.IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ● F.BF.1c. (+) Compose functions. ● F.BF.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. ● F.BF.4. Find inverse functions. ○ F.BF.4a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. ● F.LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions. ○ F.LE.1a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ● F.LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a		

relationship, or two input-output pairs (include reading these from a table). ● F.LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ● F.LE.4. For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Interpret expressions for functions in terms of the situation they model ● F.LE.5. Interpret the parameters in a linear or exponential function in terms of a context. ● F.TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. ● F.TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. ● F.TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. ● F.TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. ● F.TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ ● F.TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. ● F.TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using		

technology, and interpret them in terms of the context.★ ● F.TF.8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. ● F.TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. ● S.ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ○ S.ID.6a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.		
Content Area Literacy Standards		***21st Century Skills***
● RH.11-12.3 Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain. ● RH.11-12.4 Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines *faction* in *Federalist* No. 10). ● WHST.11-12.1 Write arguments focused on discipline-specific content. ● WHST.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.		● ***Solve Problems*** ● ***Communicate clearly*** ● ***Collaborate with others*** ● ***Be self-directed learners*** ● ***Reason effectively***

Algebra 2 Unit 3: Conics

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

Content Standards:

- A.SSE.1. Interpret expressions that represent a quantity in terms of its context.
  - A.SSE.1a. Interpret parts of an expression, such as terms, factors, and coefficients.
- A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
  - A.SSE.3a. Factor a quadratic expression to reveal the zeros of the function it defines.
  - A.SSE.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- A.APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A.APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the

Transfer

Students will be able to independently use their learning to use different representations and relationships between objects to solve real-world

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- The characteristics of quadratic relations and their representations are useful in solving real-world problems.

ESSENTIAL QUESTIONS

- How do quadratic relations model real-world problems and their solutions?

Acquisition

Students will know…

- the standard form of parabolas, circles, ellipses, and hyperbolas
- the reflective properties of parabolas, ellipses, and hyperbolas
- the center of a circle by writing the equation
- that by completing the square the equation of a circle can be solved
- the major/minor axes of ellipses and hyperbolas
- that eccentricity is used to find similarity among ellipses and hyperbolas
- that if two given points, the foci, a hyperbola is the locus of points such that the difference between the distance to each focus is constant

Students will be skilled at…

- interpreting expressions that represent a quantity in terms of its context.
- interpreting parts of an expression, such as terms, factors, and coefficients.
- choosing and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- factoring a quadratic expression to reveal the zeros of the function it defines.
- completing the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- understanding that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- identifying zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- knowing and applying the Binomial Theorem for the expansion of $(x + y)n$ in powers of x and y for a positive integer n, where x and y

zeros to construct a rough graph of the function defined by the polynomial. ● A.APR.5. (+) Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. ● A.REI.4. Solve quadratic equations in one variable. ● A.REI.4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. ● A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ ● F.IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). ● F.IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. ● F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ ○ F.IF.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	***vocabulary:*** Hyperboloid, Parabolic, Hyperbola, Parabola, Elliptical, Ellipse, Hyperbolic, Curve, Elliptic Conic Section, Circular, Circle, asymptotes, foci, major/minor axes, eccentricity, reflective properties, completing the square, center, radius, diameter, focus, directrix, tangent lines	are any numbers, with coefficients determined for example by Pascal's Triangle. ● solving quadratic equations in one variable. ● using the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. ● explaining why the x-coordinates of the points where the graph of the equation y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); ● finding the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ ● realizing that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). ● using function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. ● graphing functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ● graphing linear and quadratic functions and show intercepts, maxima, and minima. ● graphing square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ● graphing polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

○ F.IF.7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ○ F.IF.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ○ F.IF.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ○ F.IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ● F.IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. ○ F.IF.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. ● F.BF.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. ● F.BF.4. Find inverse functions. ○ F.BF.4a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. ● G.GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		● graphing rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ● graphing exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ● writing a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. ● using the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. ● identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. ● finding inverse functions. ● solving an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. ● deriving the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. ● deriving the equation of a parabola given a focus and directrix. ● deriving the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

● G.GPE.2. Derive the equation of a parabola given a focus and directrix. ● G.GPE.3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.		
Content Area Literacy Standards		***21st Century Skills***
● RH.11-12.3 Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain. ● RH.11-12.4 Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines faction in Federalist No. 10). ● WHST.11-12.1 Write arguments focused on discipline-specific content. ● WHST.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.		● ***Solve Problems*** ● ***Communicate clearly*** ● ***Collaborate with others*** ● ***Be self-directed learners*** ● ***Reason effectively***

Algebra 2 Unit 4: Probability and Statistics

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

Content Standards:

- A.SSE.1. Interpret expressions that represent a quantity in terms of its context.
  - A.SSE.1a. Interpret parts of an expression, such as terms, factors, and coefficients.
- A.REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- A.REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where

Transfer

*Students will be able to independently use their learning to **analyze dependent and independent events using varied research and reporting methods**.*

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- determining which type of event they are working with is key to formulating their statistical research.
- choosing the correct method of organizing and disseminating results, thoughts and ideas for their audience is extremely important.

ESSENTIAL QUESTIONS

- What are some considerations when undertaking a statistical study?
- How can you list the possible outcomes in the sample space of an experiment?

Acquisition

Students will know…

- that there is a difference between theoretical and experimental probabilities
- that there are relative and conditional relative frequencies and how to use conditional relative frequencies to find conditional probabilities
- that there is more than one probability rule to solve real-life problems
- The formulas for the number of permutations, and the number of combinations
- the keys needed to collect and analyze data sets
- the approximate margins of error for samples
- when it is necessary to resample
- to being the statistical process over when resampling is necessary

vocabulary: Probability experiment, Outcomes, Event, Sample space, Probability of an event, Theoretical probability, Geometric probability, Experimental probability, Independent events, Dependent events

Students will be skilled at…

- interpreting expressions that represent a quantity in terms of its context.
- interpreting parts of an expression, such as terms, factors, and coefficients.
- explaining each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- recognizing that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- explaining why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find

f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ ● F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. ● S.ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ● S.ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ● S.ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ● S.ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Summarize, represent, and interpret data on two categorical and quantitative variables ● S.ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ● S.ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ○ S.ID.6a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.	Conditional probability, Two-way table, Marginal frequencies, Joint relative frequency, Marginal relative frequency, Conditional relative frequency, Compound event, Overlapping events, Disjoint, Mutually exclusive, Permutation, n factorial, Combination, Binomial Theorem, Random variable, Probability distribution, Binomial distribution, Binomial experiment, Normal distribution, Normal curve, Standard normal distribution, z-score, Population, Sample, Parameter, Statistic, Hypothesis, Random sample, Self-selected sample, Stratified sample, Cluster sample, Convenience sample, Bias, Unbiased sample, Biases Sample, Experiment, Observational study, Survey, Simulation, Biased questions, Controlled experiment, Control group, Treatment group, Randomization, Randomized comparative experiment, Placebo, Replication, Descriptive statistics, Inferential Statistics, Margin of error	successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ● recognizing that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. ● representing data with plots on the real number line (dot plots, histograms, and box plots). ● using statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ● interpreting differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ● using the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. ● recognizing that there are data sets for which such a procedure is not appropriate. ● using calculators, spreadsheets, and tables to estimate areas under the normal curve. Summarize, represent, and interpret data on two categorical and quantitative variables ● summarizing categorical data for two categories in two-way frequency tables. ● interpreting relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). ● recognizing possible associations and trends in the data. ● representing data on two quantitative variables on a scatter plot, and describe how the variables are related. ● fitting a function to the data. ● using functions fitted to data to solve problems in the context of the data.

● S.ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ● S.ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit. ● S.ID.9. Distinguish between correlation and causation. ● S.IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ● S.IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. ● S.IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ● S.IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ● S.IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ● S.IC.6. Evaluate reports based on data. ● S.CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ● S.CP.2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ● S.CP.3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.		● using given functions or choose a function suggested by the context. ● emphasizing linear, quadratic, and exponential models. ● interpreting the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ● computing (using technology) and interpreting the correlation coefficient of a linear fit. ● distinguishing between correlation and causation. ● recognizing statistics as a process for making inferences about population parameters based on a random sample from that population. ● deciding if a specified model is consistent with results from a given data-generating process, e.g., using simulation. ● recognizing the purposes of and differences among sample surveys, experiments, and observational studies. ● explaining how randomization relates to each. ● using data from a sample survey to estimate a population mean or proportion. ● developing a margin of error through the use of simulation models for random sampling. ● using data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ● evaluating reports based on data. ● describing events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ● recognizing that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and using this characterization to determine if they are independent.

● S.CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. ● S.CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations Use the rules of probability to compute probabilities of compound events in a uniform probability model ● 5.CP.6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ● S.CP.7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. ● S.CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. ● S.CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. ● S.MD.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the ● corresponding probability distribution using the same graphical displays as for data distributions. ● S.MD.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ● S.MD.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. S.MD.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.		● recognizing the conditional probability of A given B as P(A and B)/P(B), and interpreting independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. ● constructing and interpret two-way frequency tables of data when two categories are associated with each object being classified. ● using the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. ● recognizing and explaining the concepts of conditional probability and independence in everyday language and everyday situations ● using the rules of probability to compute probabilities of compound events in a uniform probability model ● finding the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ● applying the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpreting the answer in terms of the model. ● applying the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpreting the answer in terms of the model. ● using permutations and combinations to compute probabilities of compound events and solve problems. ● defining a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. ● calculating the expected value of a random variable. ● interpreting it as the mean of the probability distribution.

● S.MD.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ○ S.MD.5a. Find the expected payoff for a game of chance. ○ S.MD.5b. Evaluate and compare strategies on the basis of expected values. ● S.MD.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ● S.MD.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game)		● developing a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated. ● finding the expected value. ● developing a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. ● weighing the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ● finding the expected payoff for a game of chance. ● evaluating and comparing strategies on the basis of expected values. ● using probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ● analyzing decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game)
Content Area Literacy Standards		***21st Century Skills***
● RH.11-12.3 Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain. ● RH.11-12.4 Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines faction in Federalist No. 10). ● WHST.11-12.1 Write arguments focused on discipline-specific content. ● WHST.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.		● ***Solve Problems*** ● ***Communicate clearly*** ● ***Collaborate with others*** ● ***Be self-directed learners*** ● ***Reason effectively***